

Isolated Vector Median Filtering for Noise Reduction in Digital Color Images

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Abstract

This paper deals with de-noising in digital color images. The conventionally used median filtering, the vector median filtering and the basic vector directional filtering and their extensions for image de-noising filter the vector pixels jointly in the red, green and blue intensities. Consequently the smoothing applied in one color component smears into the others due to the correlation preserving property of these filters. This could potentially lead to additional pixel corruption in good pixels. The recently proposed isolated vector minimum distance filtering deals by isolating the joint vectors in isolation and then minimising the aggregate distance between the pixels. This technique has proved to be much more effective for reducing noise in digital color images. This paper extends their work and proposes the isolated vector median filtering approach. The key idea of this paper is to isolate the joint vector pixels and then perform median filtering on only the most important pixels, i.e., pixels with the minimum aggregate distance from each other. We demonstrate the superiority of the proposed method compared against the aforementioned methods using several test images and statistical measures.

Keywords: Digital Image Processing, Impulse Noise, Image De-Noising, Isolated Vector Median Filter Vector Median Filters.

1. Introduction

Digital image processing has evolved over the years into a major stream in science and engineering [1 2]. An image is a two dimensional numerical data that contains the intensities of colors captured in red, green and blue. Like any electronic data, images too are subject to noise especially during transmission from one point to another. The reduction of noise (also termed image de-noising) forms a crucial component of digital image processing because it facilitates in restoring the original image and thereby reduce any information loss induced by the noise. This paper deals with de-noising of digital color images. The noise model of interest to this paper is the impulsive noise model [3]. The impulse noise is a sudden burst or slump of energy at the pixels due to switching errors or sensor temperature variations. Owing to the randomness in pixel corruption, the impulse noise model is modeled using probabilistic measures. The noise in images can be described via several application specific mathematical models. The noise model of interest to this paper is the impulsive noise model [3]. The impulse noise is a sudden burst or slump of energy at the pixels. Each pixel may be corrupted or not, and the corrupted one may be so in either of its color components. Owing to the randomness in pixel corruption, the impulse noise model can be modeled using probabilistic measures. This paper focuses on random valued impulse noise. Impulse noise is evident in several transmission systems including video and image transmission using television.

Among the several methods proposed to reduce impulsive noise in digital color images the vector median filtering method and its extensions have gained prominence due their robustness to noise [4]. The general idea of these methods is to examine and correct a test pixel's representativeness of a selected window (a set of surrounding pixels) within the image. The simplest noise reduction filter in this category is the standard median filter that computes the vector pixel median for each selected window and replaces the test pixel with the median [5]. The vector median filter (VMF) is a nonlinear approach used to process the color images as a multichannel vector [6]. This method works by replacing the test pixel by another pixel in the window that minimises a suitably chosen distance measure. The filters of this family, and especially the VMF, can perform quite robustly especially when the correlation between the three color components has to be preserved. Another version of the VMF is the basic vector directional filter (BVDF) that rather uses the angular distance between the pixels than the vector magnitudes and can be helpful when vector angles more dominant than the vector magnitudes [3, 7]. The problem however in vector median filtering is that the vector pixel is processed jointly as a multi channel. Consequently any smoothing performed therein is leveraged into all the color channels equally [8]. This could be problematic because outliers in one color component may influence the others thereby corrupting some good pixels. This problem was overcome in the recently proposed isolated vector minimum distance filter (IVMDF) which proposed a different approach to median filtering — to isolate the channels and minimise the aggregate for each processing window separately [9]. These and several methods have been used to mitigate impulse noise in television transmission [10].

This paper extends the work in [9] and proposes the isolated vector median filter. Here we put forward an important argument that minimising the aggregate distance in the window (as in [11, 3, 9]) does not necessarily ensure correct results, but computing the median of closest pixels could be beneficial especially for random noise images where the test pixel need not necessarily conform to its neighbors. The crux of the idea is to isolate the multichannels and then replace the test pixel with the computed median of a few of the most closest pixels in the processing window, as against the most closest pixel in [9]. The key merits of our proposal is improved overall noise reduction and efficient reconstruction in individual color channels. This paper argues against [4] and experimentally disproves (on impulsive noisy images) that isolated filtering introduces color artifacts.

The rest of the paper is organised as follows. The section 2 describes the signal model. This is followed by the conventional de-noising methods in section 3 and the proposed IVMF in section 4. We then present the results in section 5 and finally conclude in section 6.

2. The Signal Model

In this section, we describe the impulse noise model for color images. A digital color image can be mathematically represented as a set of joint vector pixels described as

$$X = \{x_i = 1, \dots, N\} \quad (1)$$

where $N = N_r N_c$ is the total number of pixels in the image and N_r denotes the total no of rows and N_c denotes the total number of columns. Here each pixel x_i is a 3 dimensional joint vector comprising of 3 color intensities red, green and blue as

$$x_i = \{(x_{r,i}, x_{g,i}, x_{b,i}), i = 1, \dots, N\} \quad (2)$$

Digital color images generally get corrupted by impulse noise during transmission from one source to another. If X is the original uncorrupted image, then the impulse noisy image Z at the i th pixel is described as

$$z_i = \begin{cases} x_i & \text{if } q \geq p \\ (x_{r,i}, x_{g,i}, a) & \text{if } q < p, r < 1/3 \\ (x_{r,i}, a, x_{b,i}) & \text{if } q < p, 1/3 \leq r < 2/3 \\ (a, x_{g,i}, x_{b,i}) & \text{if } q < p, r > 2/3 \end{cases} \quad (3)$$

Here $p \in (0,1]$ denotes the noise probability, i.e., the probability that the i th pixel is noisy. A large value of p indicates that a major proportion of the image is noisy and vice versa. Also $(q, r) \in (0,1]$ are continuous random numbers chosen uniformly and $a \in (0,255)$ is again chosen uniformly and denotes the noise (or corruption) of the i th pixel in the specified color component. The impulse noise model described in (3) can be summarised as follows. Each pixel is corrupted with probability p , and if the pixel is treated to be corrupted, then any one of the red, blue and green components intensity value is corrupted by a random number chosen from the interval $(0, 255)$, and the color components are chosen randomly with uniform probability. The aim of de-noising is to restore the original image X from the noisy image Z .

3. Conventional Noise Reduction Methods

In this section we describe popularly used median filter, the VMF, the BVDF and the recently propose IVMDf methods for noise reduction in digital color images.

3.1. Median filter

The median filtering was first proposed by J. W. Tukey. The filtering process considers a window (or mask) of length n (or $\sqrt{n} \times \sqrt{n}$ window), denoted as W , within the noisy image Z and then computes the median of all the pixels in the window and replaces the test pixel, usually the centre pixel, with the resulting median value. The process then repeats by sliding the window over the entire image spanning all the pixels.

Let $\{x_i, i=1, \dots, n\}$ be the vector pixels in the window W . An example window of length $n = 9$ is shown below.

An example 3×3 sliding window of length $n = 9$ is shown in figure 1.

x_1	x_4	x_7
x_2	$x_5 = x_T$	x_8
x_3	x_6	x_9

Figure 1. An example 3×3 sliding window of length $n = 9$

Each vector pixel is a 3-tuple and consists of the intensities in each of the three color components as $x_i = (x_{r,i}, x_{g,i}, x_{b,i}), i=1, \dots, n$ then the center test pixel is replaced with the Median of the window W as

$$x_T = \text{Median}(x_1, x_2, \dots, x_n) \quad (4)$$

3.2. Vector Median Filtering

The vector median filter and its extensions are derived from maximum likelihood estimation (MLE) principles of Gaussian distributions. The method is as follows. Let $x_i, i = 1, \dots, n$ be the vector pixels in the $n \times n$ window W . For each vector pixel we calculate the sum of the distances to other pixels using a distance measure. This paper uses the L_2 norm Minowski's distance measure. The distances are then added to obtain the aggregate distance

$$S_i = \sum_{j=1}^n \|x_i - x_j\|_{L=2}, i = 1, \dots, n \quad (5)$$

and then obtain the index that minimises the aggregate distance as

$$\hat{i} = \min_i \{S_i, i = 1, \dots, n\} \quad (6)$$

and then replace the test pixel with the vector pixel corresponding to the chosen index as

$$x_T = x_{\hat{i}} \quad (7)$$

It can be observed that the VMF method minimises the aggregated distance between the pixels in the text window and consequently the outliers caused due to noise are smoothed.

3.3. Basic Vector Directional Filtering

The BVD filter is a rank ordered method in which the angle between two vector pixels is used as the distance measure as against the Minowski's distance in the VMF. The method is as follows. Let $x_i, i = 1, \dots, n$ be the vector pixels in the $n \times n$ window W . For each vector pixel we calculate its angle to all other pixels and then obtain the aggregate sum as

$$\theta_i = \sum_{j=1}^n \cos^{-1} \left(\frac{x_i x_j}{\|x_i\| \|x_j\|} \right), i = 1, \dots, n \quad (8)$$

and then obtain the index that minimises the aggregate angular distance as

$$\hat{i} = \min_i \{\theta_i, i = 1, \dots, n\} \quad (9)$$

and then obtain the index that minimises the aggregate angular distance as

$$x_T = x_{\hat{i}} \quad (10)$$

It can be observed that the BVDF method minimises the aggregated angular distance between the pixels in the test window such that the directional error between the pixels is minimised. The BVDF method is particularly useful when directional processing is more appropriate and dominant in multichannel image processing.

3.4. Isolated Vector Minimum Distance Filter

The IVMDF was proposed to overcome the influence of one color component on the others. Let $x_i, i = 1, \dots, n$ be the vector pixels in the $n \times n$ window W . Each vector pixel is a

3-tuple $x_i = (x_{r,i}, x_{g,i}, x_{b,i})$. Each of these color components are isolated and then the aggregate distance of each pixel with every other pixel is computed separately for each color component as

$$d_{r,i} = \sum_{j=1}^n |x_{r,i} - x_{r,j}|, i = 1, \dots, n \quad (11)$$

$$d_{g,i} = \sum_{j=1}^n |x_{g,i} - x_{g,j}|, i = 1, \dots, n \quad (12)$$

$$d_{b,i} = \sum_{j=1}^n |x_{b,i} - x_{b,j}|, i = 1, \dots, n \quad (13)$$

and then the index that minimises the aggregate for each color component is obtained according to

$$\hat{i}_r = \min_i \{d_{r,i}\}, \hat{i}_g = \min_i \{d_{g,i}\}, \hat{i}_b = \min_i \{d_{b,i}\}$$

for $x_i, i = 1, \dots, n$ and the test pixel is replaced with those indexed pixel values as

$$x_T = (x_{r,\hat{i}_r}, x_{g,\hat{i}_g}, x_{b,\hat{i}_b}) \quad (14)$$

4. Proposed isolated vector median filter

In this paper we propose the IVMF method which is an extension to the IVMDF method. The methodology is as follows. Let $x_i, i = 1, \dots, n$ be the vector pixels in the $\sqrt{n} \times \sqrt{n}$ window W of length n . Each vector pixel is a 3-tuple $x_i = (x_{r,i}, x_{g,i}, x_{b,i})$. Each of these color components are isolated and then the aggregate Minowski's distance of each pixel with every other pixel is computed separately for each color component as in (11), (12) and (13). Whereas the IVMDF method selects that index that minimises the aggregate along each color component as in (14), this paper proposes to consider the first $L < n$ indices with the minimum aggregate and then take the median of the scalar pixel intensities corresponding to those indices, i.e., obtain the set of L indices according to

$$\{\hat{i}_{rj}\}_{j=1}^L : d_{r,1} \leq d_{r,1} \leq \dots \leq d_{r,L} \quad (15)$$

$$\{\hat{i}_{gj}\}_{j=1}^L : d_{g,1} \leq d_{g,1} \leq \dots \leq d_{g,L} \quad (16)$$

$$\{\hat{i}_{bj}\}_{j=1}^L : d_{b,1} \leq d_{b,1} \leq \dots \leq d_{b,L} \quad (17)$$

and the test pixel intensities $x_T = (x_{r,T}, x_{g,T}, x_{b,T})$ are replaced with

$$x_{r,T} = \text{Median}(x_{r,\hat{i}_{r1}}, x_{r,\hat{i}_{r2}}, \dots, x_{r,\hat{i}_{rL}}) \quad (18)$$

$$x_{g,T} = \text{Median}(x_{g,\hat{i}_{g1}}, x_{g,\hat{i}_{g2}}, \dots, x_{g,\hat{i}_{gL}}) \quad (19)$$

$$x_{b,T} = \text{Median}(x_{b,\hat{i}_{b1}}, x_{b,\hat{i}_{b2}}, \dots, x_{b,\hat{i}_{bL}}) \quad (20)$$

It can be understood from the proposed method that we do not plainly minimise the aggregate distance in the test window but instead compute the median which smooths out any outliers. The key merit of this proposal is improved smoothing which consequently

leads to improved noise reduction. Moreover because the proposed method works on each color component in isolation, any smoothing effect in one component does not drastically effect the others.

5. Experimental Study and Discussion

In this section we compare the proposed IVMF method with the state-of-the-art vector median filtering methods. We employ three measures to test the methods. The first is the root mean square error (RMSE) defined as

$$RMSE(X, Y) = \sqrt{\frac{1}{N} \sum_i \|X_i - Y_i\|^2} \quad (21)$$

where X, Y respectively are the original and filtered images. A small value of the RMSE is desirable because it indicates that the error between the filtered image and the original is small. The second measure is the peak signal-to-noise ratio (PSNR) defined as

$$PSNR = 10 \log_{10} \frac{Max(X)^2}{MSE(X, Y)} \quad (22)$$

where $MSE(X, Y)$ is the mean square error of the filtered image. A high PSNR is desirable because it indicates good signal recovery from the noise. The third measure is the structural similarity index (SSIM) defined as

$$SSIM(X, Y) = \frac{(2\mu_x\mu_y + c_1)(2C_{x,y} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \quad (23)$$

where the image means are

$$\mu_x = \frac{1}{N} \sum_{i=1}^N X_i \quad (24)$$

$$\mu_y = \frac{1}{N} \sum_{i=1}^N Y_i \quad (25)$$

and $\sigma_{(.)}^2$ denotes the variance and $C_{(.,.)}$ denotes the covariance between the original and filtered images. The SSIM value denotes the similarity between two images, the original and the filtered in our case, by incorporating perceptual features including luminance and contrast. A high value of SSIM indicates accurate reconstruction of the original image. An overview of these comparative measures can be found in [12]. The Matlab code to implement the aforementioned state-of-the-art methods and the proposed method of image denoising can be downloaded from this [clickable link](#).

We first demonstrate the superiority of the proposed IVMF method over state-of-the-art filtering methods. In Figure 2 we display the test case image used for this purpose. For this test case, Figure 3 shows the noisy image and the filtered images at varying noise probabilities and it can be visually observed that the last column corresponding to the proposed IVMF method outperforms the other methods.



Figure 2. An example test image

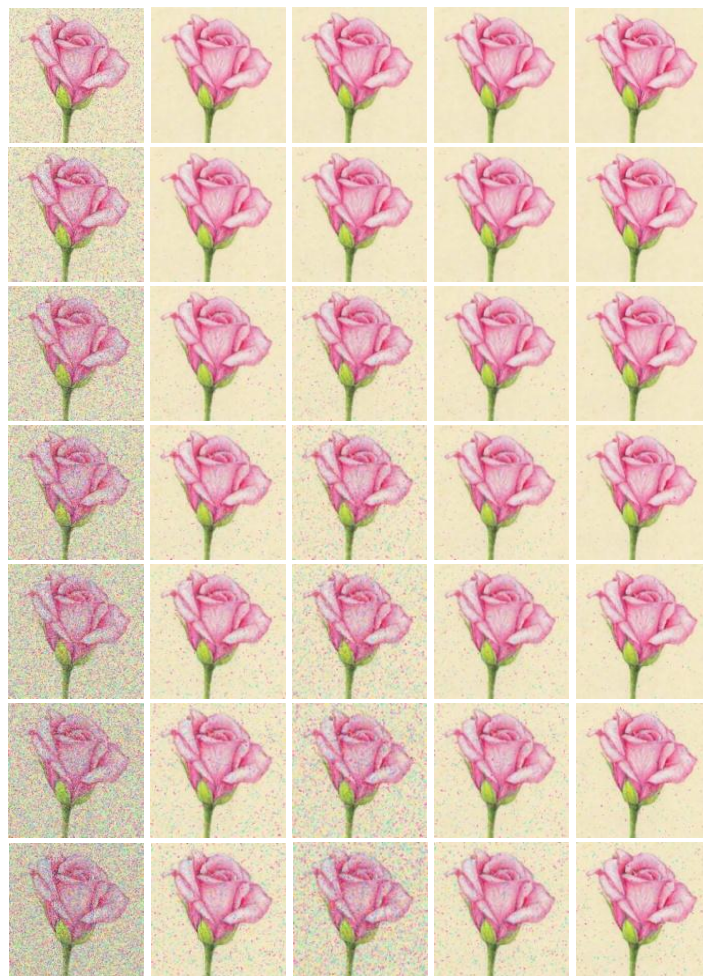


Figure 3. Top to bottom: $p = 0.3, 0.4, 0.6, 0.7, 0.8, 0.9, 0.99$. Left to right: (a) noisy image, (b) median filter, (c) VMF, (d) IVMDF, (e) Proposed IVMF ($L = 6$)

For the demonstration in Figure 3, the RMSE, the PSNR and the SSIM performance versus the noise probability is shown in Figures 4, 5 and 6 respectively. We show our method for values of $L = 4, 6$. It can be seen that our IVMF method shows improved accuracy (i.e., low RMSE) than that of the joint vector approaches and also the IVMDF approach and also performs outstandingly well in high noise. While the IVMF method

with $L = 6$ is 1.2 times superior to the median filter at $p = 0.5$, it is nearly twice better at $p = 0.99$ indicating that the methods scales well in high noise conditions. This can be attributed to the fact that the skewness is dealt with for each color component separately, thus leading to a more accurate reconstruction. Moreover our method exhibits about 84% improvement over the median filter and 50% over the IMVDF in terms of PSNR at $p = 0.99$. These results in summary demonstrate the superiority of the proposed method in noise mitigation. Moreover the proposed method at $L = 6$ has shown a consistent edge of nearly 10% over the case at $L = 4$ in both RMSE and PSNR tests indicating that taking the median of the closest pixels is desirable than considering the single pixel that minimises the aggregate.

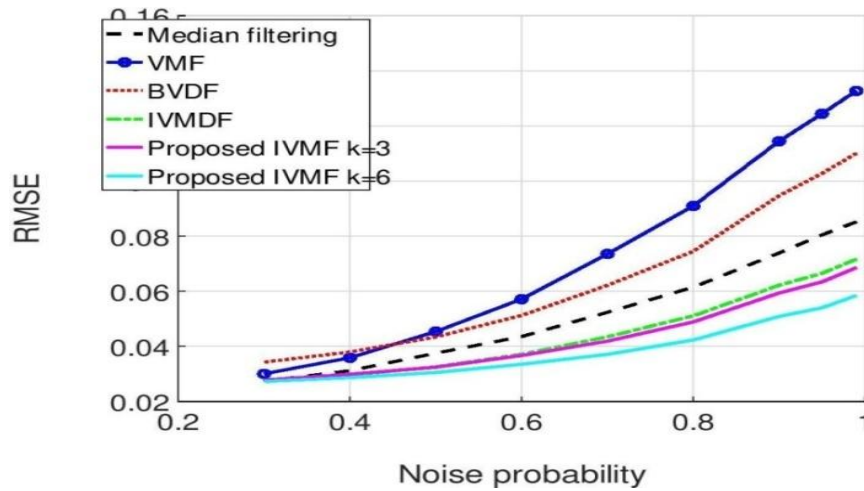


Figure 4. The RMSE versus the noise probability.

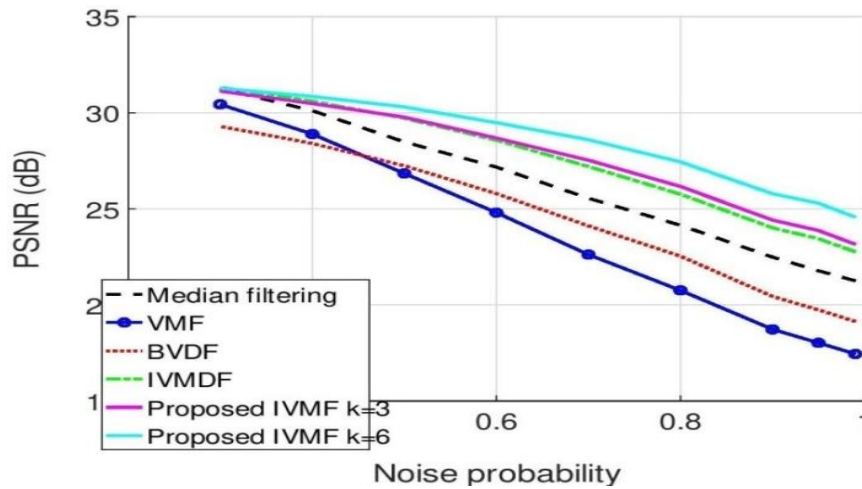


Figure 5. The PSNR versus the noise probability.

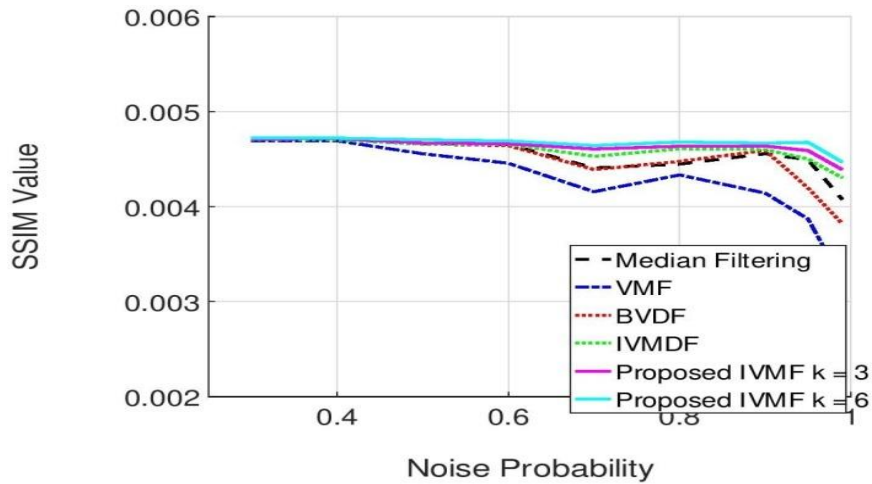


Figure 6. The SSIM values versus the noise probability.

What is outstanding is that the proposed method performs exceedingly well in terms of recovering the structural similarity of the original image as shown in Figure 6. The IVMF method achieves this by treating each color dimension in isolation. However, the method suffers from recovering the edges accurately as compared to the VMF method as there is an inherent loss of color mixing within the filtering technique. In spite of this limitation, the proposed method remains superior in terms of restoring the perceptual structure of the filtering image as shown in Figure 7. This figure depicts the ratio of the SSIM value of the proposed IVMF method to the SSIM value of the next best performing method in terms of RMSE and PSNR, the standard median filter for varying noise probabilities, and clearly shows that the proposed method scales nearly exponentially with noise when compared against the standard median filter. This paper thus brings to light that the simple and straightforward idea of isolating the color components and then performing vector median filtering leads to substantial improvement in noise mitigation.

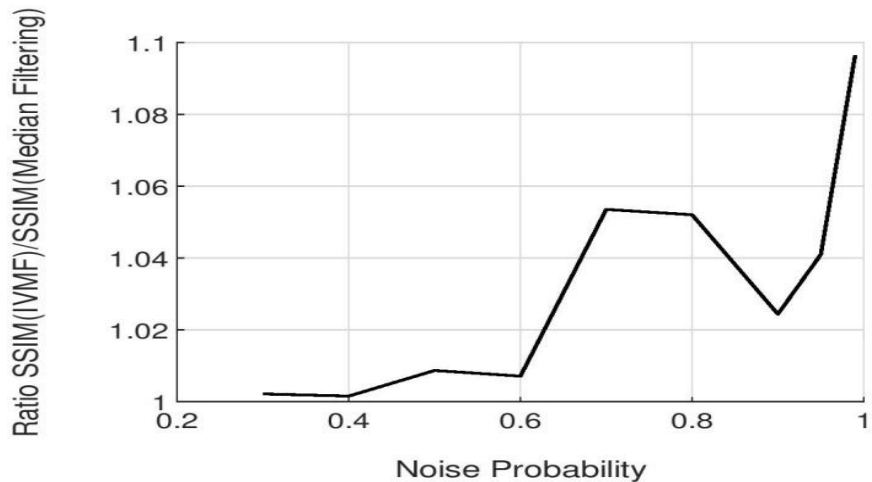


Figure 7. The ratio of SSIM value of the proposed IVMF method to the SSIM value of the standard median filtering method versus the noise probability.

The merit of filtering the color components in isolation is now shown in Figure 8. For this we use the test case original image which is the first image in the first row of the figure. The impulse noise impaired version at $p = 0.8$ is shown next to the original. The

filtered red, green and blue components are shown there under in double precision. It can be seen that when the individual color components are observed in isolation, the proposed IVMF method repairs most of the impaired pixels. This efficiency is shown in Figure 9 which shows the average percentage of pixels with error greater than 0.1. The average is taken across all the three color components. It can be

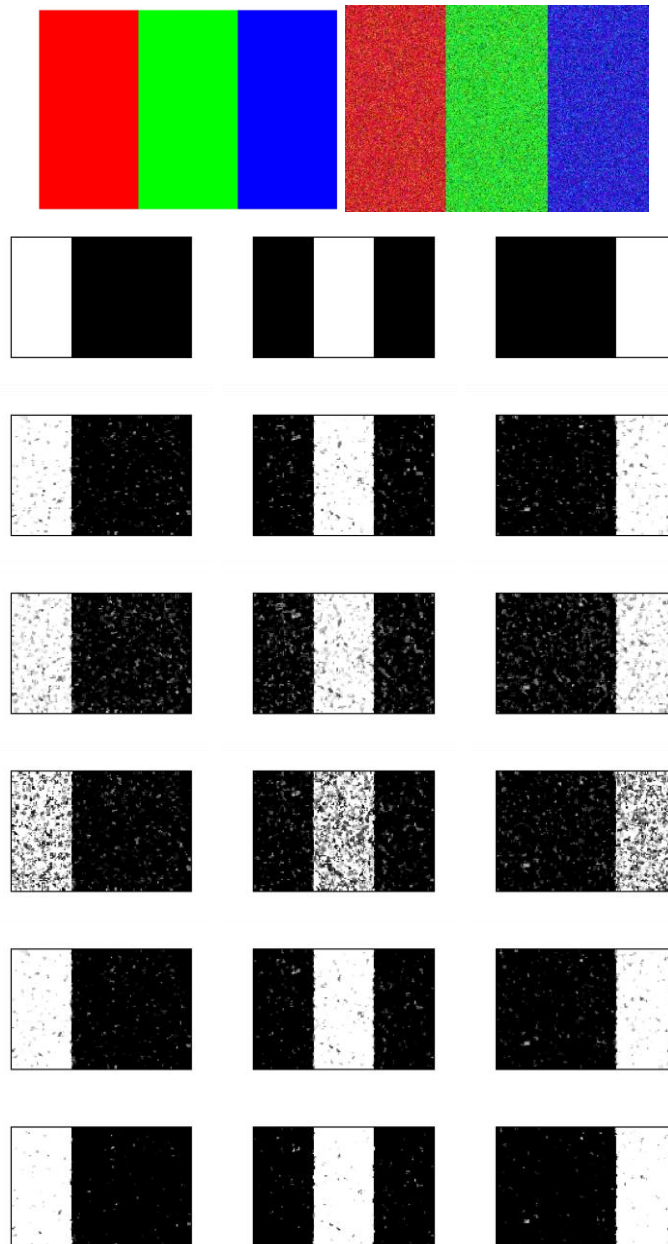


Figure 8. First row — left to right: (a) original, (b) noisy. From the second row — left to right, (a) red component, (b) green component, (c) green component in double precision mode, top to bottom: (a) median filtering, (b) VMF, (c) BVDF, (d) IVMDf, (e) Proposed IVMF $L = 3.18$

observed that the proposed IVMF and the IVMDf methods repair more pixels than the other methods. Moreover it can also be observed that the VMF and the BVDF methods, that are known to better detect and reconstruct the edges, get polynomially worse with increase in noise.

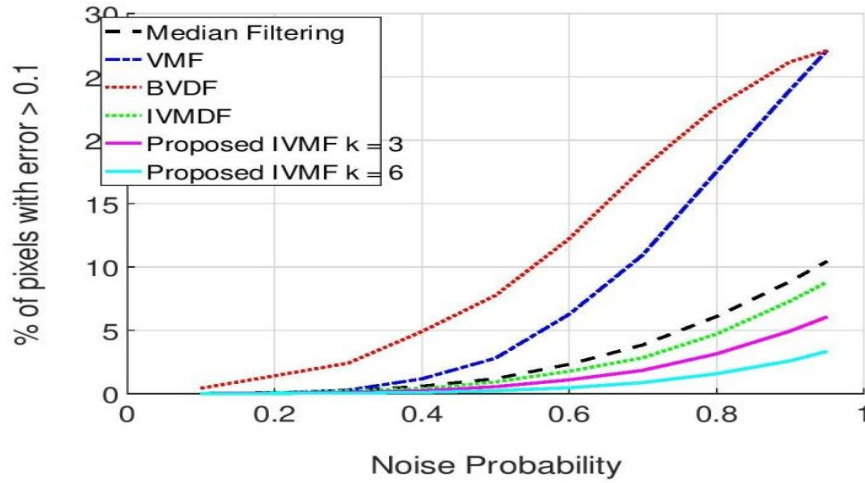


Figure 9. The average percentage of pixels with error greater than 0.1 for varying values of noise pixels.

To generalise the proposed method, we show the filtering output for more images in Figure 10 for a noise probability of $p = 0.8$ and also report the RMSE values in Table 1. It can be visually observed that the proposed IVMDF out- performs the other methods by virtue of performing minimum median distance filtering for vector pixels in isolation.

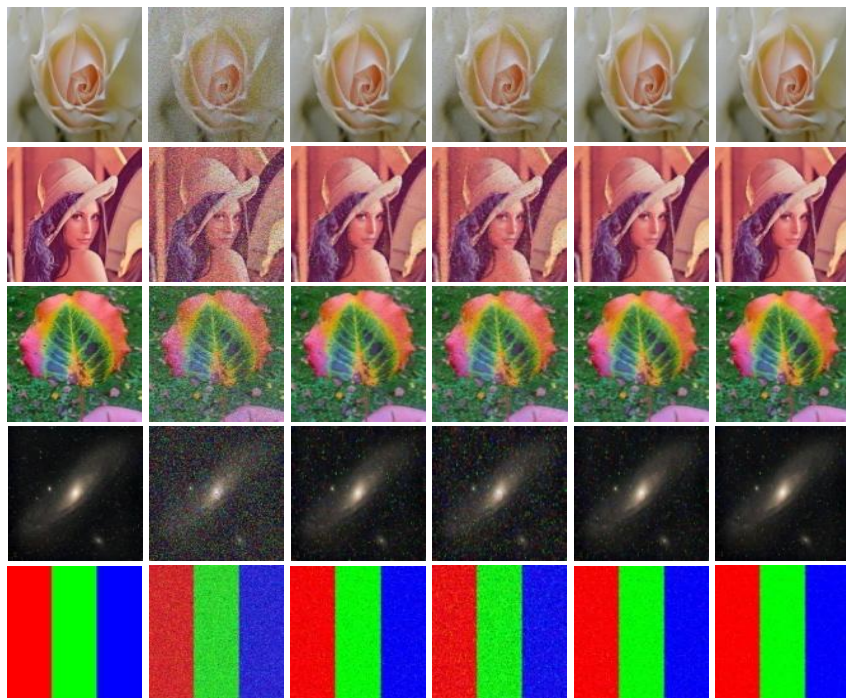


Figure 10. Left to right: (a) original, (b) noisy, (c) median (d) VMF, (e) IVMDF, (f) Proposed IVMF (L = 6)

Table 1. The table shows the RMSE values for the images shown in Figure 9 (in respective order from top to bottom). In addition to the filtering results shown in the figure we also present the RMSE values of the BVDF method.

No	Median	VMF	BVDF	IVMDF	IVMF
1	0.026603	0.047705	0.050471	0.019432	0.015269

2	0.056148	0.076522	0.091692	0.051082	0.047564
3	0.059965	0.081590	0.093800	0.054473	0.050823
4	0.067514	0.10307	0.092730	0.054685	0.044630
5	0.10085	0.16010	0.26986	0.083136	0.061291

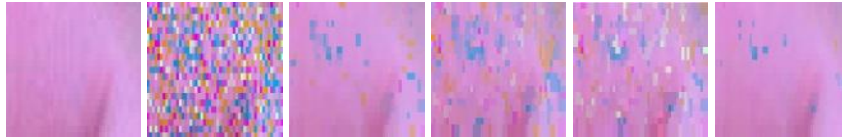


Figure 11. Images: (a) original, (b) noisy, (c) median filter, (d) VMF, (e) BVDF, (f) IVMF ($L = 6$).

Moreover it can be observed with close inspection that our proposed method (the last column) does not suffer much from color component smearing, e.g., red appearing in green, green appearing in black, etc., as does the vector median filters thus validating our claim that isolated filtering overcomes the smearing effect suffered by the joint filtering techniques due to correlation within the color components. This we demonstrate Figure 11 corresponding to the bottom right corner in the third image in Figure 10. It can be observed that the IVMF corrects more color artifacts than the joint vector median filters by virtue of leveraging the filtering process locally within each color component.

6. CONCLUSION

This paper proposed the isolated vector median filtering for noise suppression in digital color images. The method operates by isolating each of the color components and then computing the mean of a fixed number of pixel values with least distance between themselves. The key merit of this proposal is that the smoothing induced therein stays local and does not affect the other color components and hence leads to improved accuracy in image reconstruction. The down side, although, is that the proposed method does not deal effectively in denoising near the edges and this problem will be dealt in the future work. We have demonstrated the superiority of the proposed method using real images and several statistical measures.

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