

# A sorted weighting lookahead sampling scheme for accurate and fast particle filtering

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**Abstract**—The particle filter is known to be a powerful tool for estimating hidden Markov processes in nonlinear and nonGaussian state space models. The filter involves generating new particles from old ones, from regions of high importance in the state space using a proposal distribution and then weighing them using the incoming observation. However a poor choice of the proposal distribution may migrate the new particles into regions that do not contribute to the posterior and hence lead to one particle accumulating all the weight (termed particle degeneracy). This degeneracy is overcome using the resampling step that eliminates those particles with low weights and replaces them by those with large weights. However this resampling step is a computationally demanding operation. In the literature, the methods that speed up the particle filter, like the Gaussian particle filter, trade tracking accuracy with speed while methods that sample particles from high importance regions, like the auxiliary particle filters and lookahead particle filters, trade speed with accuracy. In this paper we propose a simple lookahead sampling scheme. Here the particles that fall into high importance regions are predetermined (seen ahead) and then propagated in copies to make up for those that should be discarded. This strategy avoids the resampling step and consequently leads to high speed and accuracy. Using two nonlinear models, we show the tracking efficiency of the proposed method.

**Index Terms**—Bayesian state estimation, particle filter, lookahead sampling, sorted weights, RMSE

## I. INTRODUCTION

The Bayesian state estimation is an important solution to estimate hidden dynamic target states and is used widely in target tracking applications [1], [2]. The popular Kalman filter provides an optimal estimate of the state of the target and the associated uncertainty for Bayesian estimation in linear Gaussian systems [3]. However for nonlinear and nonGaussian systems, a closed form solution cannot be derived due to intractable integrals involved in the estimation process. This intractability is overcome using approximate solutions. An efficient Bayesian approximation is the particle filter (PF) which approximates the posterior probability density function (pdf) of the target state by a set of weighted particles [4], [5]. The particles within the PF can be understood as weighted point explorers that localise themselves near the regions of high importance, i.e., the regions that contribute

to the posterior. The first step in the PF is the sequential importance sampling (SIS) that specifies the process of sampling new particles at each time step from the previous ones and updating their weights. The SIS step can sample particles from regions of high importance only when it leverages the incoming observation within the sampling process. However this leveraging is difficult and not straightforward. Therefore SIS by itself, encounters degeneracy, a problem in which, after a few iterations, all but one particle have negligible weights, which is a direct consequence of not sampling from high importance regions. This is overcome in the second step, the resampling, that eliminates particles with negligible weights and replaces them by those with large weights [6], [7].

Several methods have been proposed to predict particles from regions of high importance by conditioning the incoming observation in the SIS process. The key idea of these methods is to sample a set of particles using the previous ones, weigh them using the incoming observation and then use the weights to sample a final set of weighted particles. The most popular method in this category is the auxiliary particle filter (APF) [8]. The filter samples a lookahead set of particles and computes their weights, then resamples the lookahead set and uses those resampling indices to propagate the old particles to the next time step. The recently proposed improved APF (IAPF) described the APF within the multiple importance sampling framework and proposed a general framework to compute the weights from the lookahead set of particles [9], [10]. The IAPF has shown improved tracking accuracy over the APF, especially in low noise scenarios. Other lookahead strategies include the adapted placement and others [11], [12]. These methods achieve high tracking accuracy by virtue of sampling good particles. However they involve the computationally demanding resampling step and additional sampling and weighing steps and hence trade speed for accuracy.

The key property of the PF is that using a large number of particles will ensure them being sampled from regions of high importance and hence the weighted particle approximation will

approach the true posterior pdf. However the resampling step in the particle filter is a computationally demanding sequential operation and thus prohibits the use of a large number of particles [13]. The recently proposed fast resampling approaches — the Metropolis [14], distributed computing [15] and the random network [16] resampling methods accelerate the PF by reducing the communication within the particles. However this reduction in inter-particle communication deteriorates the tracking performance. The Gaussian PF totally alleviates the need for resampling as it propagates only the first and second moments of the Gaussian densities using weighted samples [17]. However the filter is limited to additive Gaussian systems and tracks poorly when the diffusion over the state transition density is large. Overall these methods trade accuracy for speed.

In this paper, we propose a simple lookahead particle filter that is computationally efficient and operationally accurate. The key idea here is to lookahead in time by sampling particles and weighing them using the incoming observation. This lookahead set is then sorted in accordance to its weights. Then a few large weight particles (that lie in regions of high importance) are deterministically selected to be propagated forward. This deterministic selection avoids the need to resample and hence the method gains tremendously in speed. These propagated particles ensure good tracking accuracy as the new lookahead SIS leverages the observation in the propagation step.

The rest of the paper is organised as follows. Sections II and III sets the notation and presents the Bayesian estimation and PF methods. This is followed by the proposed weighted sorting lookahead PF in section IV and evaluation results in section V. We finally conclude in section VI.

## II. BAYESIAN ESTIMATION

The target state (or signal) of interest  $\mathbf{x}_t \in \mathbb{R}^{d_x}$  at time instant  $t \in \mathbb{N}$  is a hidden Markov process with initial distribution  $p(\mathbf{x}_{t=0})$  and the Markov state transition  $p(\mathbf{x}_t|\mathbf{x}_{t-1})$  for time steps  $t = 1, \dots, T$  with  $d_x$  denoting the dimensionality of the target. The sensor observations  $\mathbf{y}_t \in \mathbb{R}^{d_y}$  are conditionally independent given the state variable  $\mathbf{x}_t$  and is given by the observation density  $p(\mathbf{y}_t|\mathbf{x}_t)$  where  $d_y$  denotes the dimensionality of the observation. The set of states and the observations are denoted as  $\mathbf{x}_{1:t} = \{\mathbf{x}_1, \dots, \mathbf{x}_t\}$  and  $\mathbf{y}_{1:t} = \{\mathbf{y}_1, \dots, \mathbf{y}_t\}$ . This target state transition and sensor observation models together constitute the discrete time state space model given by

$$\mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{a}_t) \quad (1)$$

$$\mathbf{y}_t = h(\mathbf{x}_t, \mathbf{e}_t) \quad (2)$$

where the (possibly) nonlinear functions  $f(\cdot)$  and  $h(\cdot)$  respectively are the state transition and sensor observation functions and  $\mathbf{a}_t$  and  $\mathbf{e}_t$  respectively are the state transition and observation noise.

The aim of Bayesian state estimation is to estimate sequentially in time the pdf of the hidden target state using all

available observations. In the Bayesian context, if the posterior pdf  $p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})$  at time  $t-1$  is available, then the aim is to recursively estimate a state prediction pdf  $p(\mathbf{x}_t|\mathbf{y}_{1:t-1})$  using (1) and a posterior pdf  $p(\mathbf{x}_t|\mathbf{y}_{1:t})$  using (2). Using these densities the target state can be estimated for any given model parameters. This Bayesian recursion is given by

$$\underbrace{p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})}_{\text{posterior at time } t-1} \longrightarrow \underbrace{p(\mathbf{x}_t|\mathbf{y}_{1:t-1})}_{\text{prediction at time } t} \longrightarrow \underbrace{p(\mathbf{x}_t|\mathbf{y}_{1:t})}_{\text{updated posterior at time } t} \quad (3)$$

where the predicted and updated pdfs are

$$p(\mathbf{x}_t|\mathbf{y}_{1:t-1}) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1})p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1}) d\mathbf{x}_{t-1} \quad (4)$$

$$\begin{aligned} p(\mathbf{x}_t|\mathbf{y}_{1:t}) &= \frac{p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})}{p(\mathbf{y}_t|\mathbf{y}_{1:t-1})} \\ &\propto p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1}) \end{aligned} \quad (5)$$

Once the updated pdf is available, the hidden target state can be estimated using the expected *a posteriori* (EAP) [2] as

$$\hat{\mathbf{x}}_t^{\text{EAP}} = \mathbb{E}(p(\mathbf{x}_t|\mathbf{y}_{1:t})) = \int \mathbf{x}_t p(\mathbf{x}_t|\mathbf{y}_{1:t}) d\mathbf{x}_t \quad (6)$$

## III. PARTICLE FILTERING METHODS

In this section we briefly describe the conventionally used PF methods. The posterior pdf of the target state  $p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})$  at time  $t-1$  is represented by a set of particles and their corresponding weights  $\{\mathbf{x}_{t-1}^i, w_{t-1}^i\}_{i=1}^N$ , where  $i$  is the particle index and  $N$  is the total number of particles. This weighted set representation is given by the sum of weighted delta functions as

$$p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1}) \approx \sum_{i=1}^N w_{t-1}^i \delta(\mathbf{x}_{t-1} - \mathbf{x}_{t-1}^i) \quad (7)$$

At time step  $t$ , the PF generates a new set of particles from the old ones using a proposal distribution as

$$\mathbf{x}_t^i \sim q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t), \quad i = 1, \dots, N \quad (8)$$

The new particles are now representative of the predicted pdf in (4) as

$$p(\mathbf{x}_t|\mathbf{y}_{1:t-1}) \approx \sum_{i=1}^N w_{t-1}^i \delta(\mathbf{x}_t - \mathbf{x}_{t-1}^i) \quad (9)$$

The new particles are then weighted as

$$w_t^i = w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i)p(\mathbf{x}_t^i|\mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t^i|\mathbf{x}_{t-1}^i, \mathbf{y}_t)} \quad (10)$$

$$\propto w_{t-1}^i p(\mathbf{y}_t|\mathbf{x}_t^i), \quad i = 1, \dots, N \quad (11)$$

where (11) follows by taking a convenient assumption that the particles are drawn from the state transition density as  $q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)$ . This normalised weighted set of particles is representative of the posterior pdf at time  $t$  as

$$p(\mathbf{x}_t|\mathbf{y}_{1:t}) \approx \sum_{i=1}^N w_t^i \delta(\mathbf{x}_t - \mathbf{x}_t^i) \quad (12)$$

After a few iterations, the discrepancy between the weights increases, leading to degeneracy. The solution to this is the resampling step wherein those particles that have negligible weights are replaced by exact copies of other particles that have larger weights, i.e., for  $i = 1, \dots, N$ , we sample an index  $j(i)$  distributed according to the probability  $P(j(i) = m) = w_t^m, m = 1, \dots, N$  and replace  $\mathbf{x}_t^i = \mathbf{x}_t^{j(i)}$  and set  $w_t^i = 1/N$ .

The proposal density  $q(\mathbf{x}_t | \mathbf{x}_{t-1}^i, \mathbf{y}_t)$  in (8) aims to draw particles from the regions of importance. This facilitates the particles to lookahead in time using  $\mathbf{y}_t$  and explore the right state space and thus avoid particle degeneracy. That is, the better the proposal density, the better placed the particles and the lesser the need to resample. However it is difficult to leverage the observation  $\mathbf{y}_t$  directly in the proposal  $q(\cdot)$ . Several lookahead strategies have been proposed to this effect. The most popular is the APF and its variants that accomplishes lookahead filtering as follows. Here the particles are propagated forward as  $\mathbf{v}_t^i = \mathbb{E}(\mathbf{x}_t | \mathbf{x}_{t-1}^i)$  and weighted according to  $\bar{w}_t^i = w_{t-1}^i p(\mathbf{y}_t | \mathbf{v}_t^i)$ . Resampling this normalised weighted lookahead set to obtain a set of indices  $j(i)$  for  $i = 1, \dots, N$  such that  $P(j(i) = m) = \bar{w}_t^m$  will indicate that set of particles at time  $t-1$  whose propagation to time  $t$  will place them in regions of high importance. Hence the new particles will be generated as in (8) as

$$\mathbf{x}_t^i \sim p(\mathbf{x}_t | \mathbf{x}_{t-1}^{j(i)}), i = 1, \dots, N \quad (13)$$

Following (10) the weights are updated as

$$w_t^i = \frac{p(\mathbf{y}_t | \mathbf{x}_t^i)}{p(\mathbf{y}_t | \mathbf{v}_t^{j(i)})} \quad (14)$$

and normalised. The denominator compensates for the lookahead proposal described earlier. The recently proposed IAPF generalises the APF scheme and has demonstrated improved performance.

These lookahead schemes generate particles from regions of high importance thus leading to improved accuracy but fail to avoid the computationally intensive resampling step. On the other hand, the fast resampling PFs suffer from reduced tracking accuracy as they limit the communication within the particles. In the next section, we propose a simple lookahead PF that overcomes these two problems.

#### IV. PROPOSED WEIGHT-SORTED LOOKAHEAD PF

Consider a set of particles  $\{\mathbf{x}_{t-1}^i, w_{t-1}^i\}_{i=1}^N$  representing the posterior at time  $t-1$ . These particles are propagated to time  $t$  using the state transition density as  $\mathbf{v}_t^i \sim p(\mathbf{x}_t | \mathbf{x}_{t-1}^i)$  and then weighted according to  $a_t^i \propto w_{t-1}^i p(\mathbf{y}_t | \mathbf{v}_t^i)$  for  $i = 1, \dots, N$ . Then these unnormalised weights are sorted from the largest to the smallest and the particles are also rearranged according to the sorted weights to obtain the sorted set

$$\{\mathbf{v}_t^i, a_t^i\}_{i=1}^N : a_t^{i-1} \geq a_t^i, i = 1, \dots, N \quad (15)$$

These particles weighted in accordance to the observation are termed the *lookahead particles* as they will be used to obtain

a final set of particles placed in regions of high importance. We then compute the ratios

$$\gamma_t^i = a_t^i / a_t^1, i = \dots, N \quad (16)$$

It can be observed that the vector  $\gamma_t^{1:N}$  will be a monotonically decreasing function. We then deterministically select those set of indices whose sorted-weight-ratio is greater than a certain threshold  $T_h$  according to

$$\eta_t = \{j = 1, \dots, M : \gamma_t^j \geq T_h\} \quad (17)$$

It is apparent that the first  $M$  weighted particles are indicative of being located in regions of high importance and the remaining  $N - M$  are not. Therefore the  $N - M$  small weight particles will be eliminated. For this we obtain an index vector

$$\{j(i)_{i=1, \dots, N}\} = \left\{ \underbrace{\eta_t, \dots, \eta_t}_{\lfloor N/M \rfloor \text{ times}}, \eta_t \left( 1, \dots, N - M \left\lfloor \frac{N}{M} \right\rfloor \right) \right\} \quad (18)$$

such that the first  $M$  indices in the set  $\eta_t$  are replicated until the cardinality  $|\{j(i)_{i=1, \dots, N}\}| = N$ . The final weighted particles are obtained, analogous to (8), by setting

$$\mathbf{x}_t^i = \mathbf{v}_t^{j(i)}, w_t^i = a_t^{j(i)}, i = 1, \dots, N \quad (19)$$

The condition that  $N = M$  implies that all the particles lie within a region that contributes to the posterior and therefore they can be straightforwardly retained. When  $M < N$ , the  $N - M$  particles and their weights in the sorted set in (15) will be eliminated. To fill for the eliminated particles and weights, the retained  $M$  particles are replicated  $\lfloor N/M \rfloor$  times and the remaining  $\beta = N - M \lfloor N/M \rfloor$  are replicated by the first  $\beta$  particles in the sorted set, to fill the set with  $N$  weighted particles.

It can be understood that this selection and replication scheme in (18) and (19) leverages on the incoming observation and fills the particle set with high weight particles such that all of them lie in regions of high importance. However, the condition  $M \ll N$ , at any time step, will lead to too many replications and thereby loss of particle diversity (conventionally termed sample impoverishment [5]). To avoid this, we propose to perturb the  $M + 1$  to  $N$  new particles as

$$\mathbf{x}_t^i = \mathbf{x}_t^i + \mathcal{N}(0, \epsilon), i = M + 1, \dots, N \quad (20)$$

where  $\epsilon \ll \ll$ . This ensures richness in the particles while propagating them to regions of high importance. When the  $N - M$  particles are perturbed their corresponding weights may be recalculated as

$$w_t^i = \frac{p(\mathbf{y}_t | \mathbf{x}_t^i)}{p(\mathbf{y}_t | \mathbf{v}_t^{j(i)})}, i = M + 1, \dots, N \quad (21)$$

The final normalised weighted particle set  $\{\mathbf{x}_t^i, w_t^i\}_{i=1}^N$  now represents the posterior at time  $t$  and the state estimate can be calculated from (6) as  $\hat{\mathbf{x}}_t = \sum_{i=1}^N w_t^i \mathbf{x}_t^i$ . The choice of  $T_h$  becomes critical to the performance of the method. For now, the value is chosen empirically and set to  $T_h \in (0.001, 0.1)$

based on the model of study. The key merits of this proposal is that we completely bypass the need to resample and instead deterministically chose a set of particles for propagation; the only additional computation comes from sorting with order of complexity  $O(N)$  and recalculating the  $N - M$  weights. Moreover the proposal achieves good tracking accuracy by virtue of sampling from the regions of high importance.

## V. EVALUATION STUDY

In this section we use two nonlinear models to evaluate the performance of the proposed method in terms of the root mean square error (RMSE) and computational time (in seconds). Consider the nonlinear state space model given by

$$x_t = \frac{x_{t-1}}{2} + \frac{25x_{t-1}}{1+x_{t-1}^2} + 8\cos(1.2t) + a_t$$

$$y_t = \tan^{-1}(x_t) + e_t$$

where the state transition noise and the observation noise respectively are  $a_t \sim \mathcal{N}(0, \tau^2)$  and  $e_t \sim \mathcal{N}(0, \sigma^2)$ . We compare the proposed sorted-weighting lookahead PF (WSLAPF) with the standard PF, the APF, the IAPF and the GPF. The total number of time steps are  $T = 200$ , the initial distribution is  $p(\mathbf{x}_{t=0}) = \mathcal{N}(5, 1)$ ,  $\tau^2 = 5$  and the results are averaged over 1000 Monte Carlo runs. In Fig. 1 we present the RMSE versus the observation error precision with the number of particles  $N = 100$  and the WSLAPF threshold set to  $T_h = 0.01$ . Fig. 2 shows the computational time versus the number of

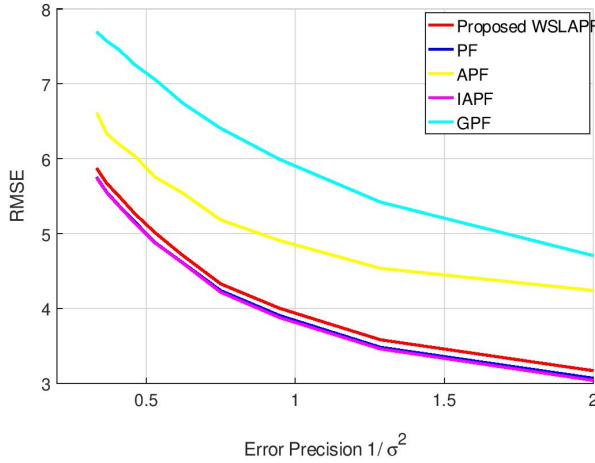


Fig. 1. The RMSE versus the error precision  $1/\sigma^2$ .

particles with observation noise variance  $\sigma^2 = 1$ . By observing both the figures, it can be seen that the proposed method compares favourably with the traditional PFs in accuracy and demonstrates tremendous computational advantage by virtue of bypassing the need to resample every time step. The GPF, albeit being the fastest among all, suffers in tracking accurately and this inaccuracy is more pronounced when the transition density diffusion over the states determined by  $\tau^2$  is large as chosen for this example. The APF, as referred in

[10] tracks poorly in low noise scenarios, as chosen in this example. The IAPF demonstrates good tracking accuracy but is computationally more demanding owing to its additional weighing and resampling steps.

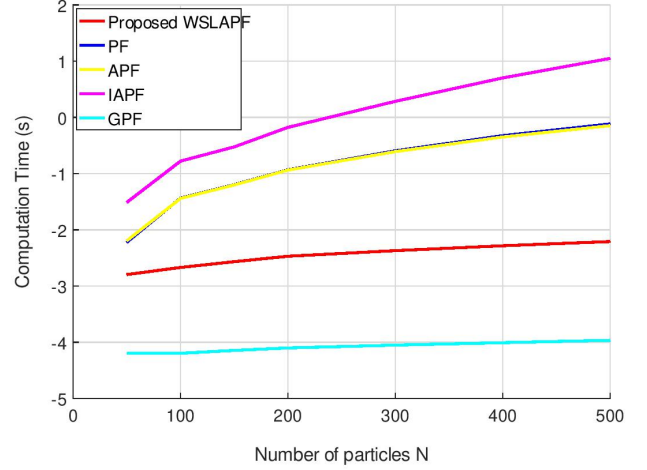


Fig. 2. The computational time versus the number of particles  $N$ .

To put this time-error comparison in full perspective, we plot the time-scaled-RMSE (TxRMSE) (Computational time  $\times$  RMSE) for  $N = 100$  versus the error precision in Fig. 3. It can be observed that the proposed method shows tremendous gain over the existing PFs and its TxRMSE does not increase polynomially in high noise conditions. The proposed method exhibits nearly 2 times improvement over the PF and the APF and 5 times over the IAPF. Although the GPF shows the best TxRMSE, the filter is limited to additive Gaussian models only. We now show in Fig. 4 the effect of the choice of

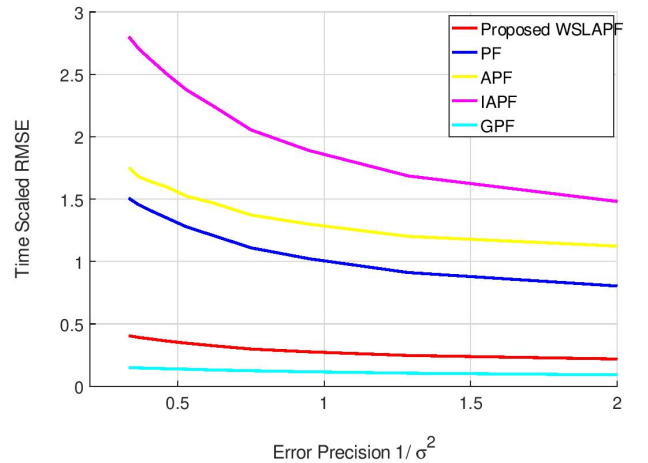


Fig. 3. The time-scaled-RMSE versus the error precision  $1/\sigma^2$ .

the threshold on the tracking performance with  $\sigma^2 = 1$  and  $N = 100$ . Low  $T_h \lll$  will lead to small weights gathering

into the particle set and high  $T_h \gg \gg$  with lead to sample impoverishment. It can be observed that there is an optimal minimum for  $T_h \in (0.02, 0.04)$  and that a carefully chosen small value would ensure good tracking performance.

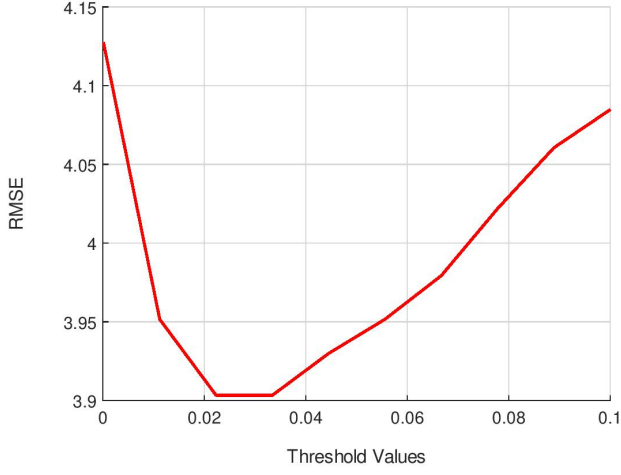


Fig. 4. The RMSE versus the threshold  $T_h$ .

We now evaluate the proposed method on a 1-D image observation model for tracking a single moving target. This example is critical for many tracking applications using radar/sonar/image data. The target state follows a random walk as  $\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{a}_t$ ,  $\mathbf{a}_t \sim \mathcal{N}(0, \tau^2)$ . The observation model, as described in [18], is given by

$$\begin{aligned} \mathbf{y}_t(j) &= \mathbf{h}_t(j) + \mathbf{e}_t, e_t \sim \mathcal{N}(0, \sigma^2) \\ &= \frac{I \Delta s(j)}{2\pi\Sigma} \exp\left(-\frac{(s(j) - \mathbf{x}_t)^2}{2\Sigma}\right) + \mathbf{e}_t, j = 1, \dots, K \end{aligned}$$

where  $s(j)$  is the  $j$ th pixel in the image having a total of  $K$  pixels. The image observes a static region of surveillance defined by a grid as  $s(j) \in (s_{\min}, s_{\max})$  with width of each grid point being  $\Delta s(j) = 1$ . We interpret  $\mathbf{h}_t(j)$  as the contribution of the target  $\mathbf{x}_t$  to the intensity at the  $j$ th pixel and blurred by a factor of  $\Sigma$ .

In Fig. 5 we illustrate the validity of our proposal using a simulation for  $T = 100$  time steps and the number of particles  $N = 200$ . The image observations are shown on the top panel and the filter estimates are shown on the bottom. It can be seen that the image observations are noisy (the error variance  $\sigma^2 = 0.2$ ) so much so it is difficult to detect the target by looking at one observation as depicted in Fig. 6. The actual target follows a sinusoidal path. It can be observed that the proposed PF and the conventional PFs track the target accurately and also maintains lock during high manoeuvres.

We now test the time-scaled-RMSE of the proposed method for the nonlinear image observation model for  $T = 100$  time steps (the APF and the GPF are omitted in this example for convenience). The number of particles is  $N = 200$ . The

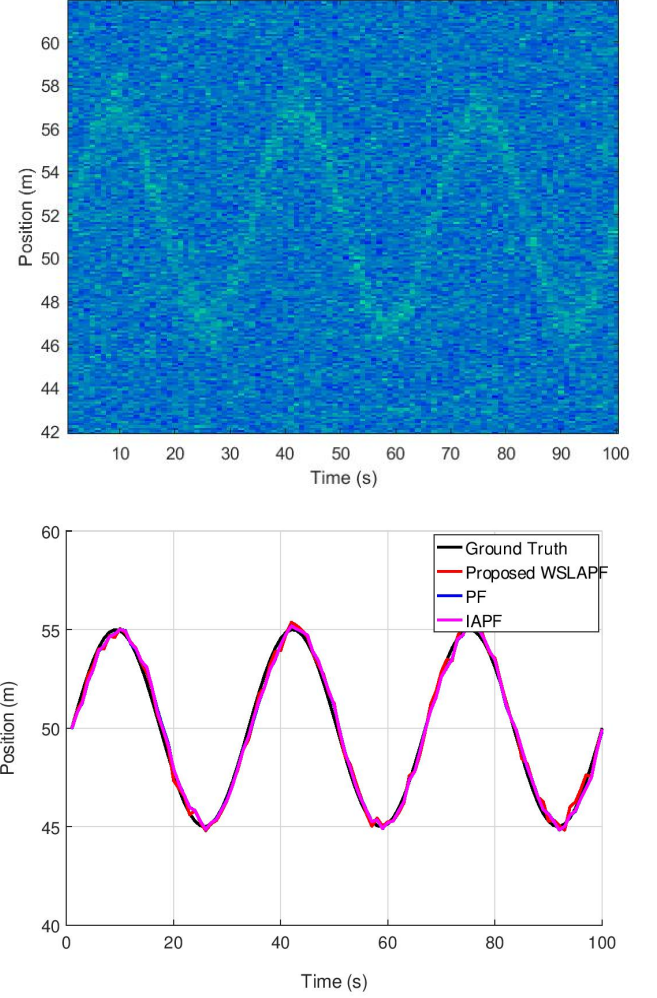


Fig. 5. Illustration of the tracking of a highly manoeuvring target from noisy 1-D image observations. The top panel shows the observations over time and the bottom panel shows the filter estimates.

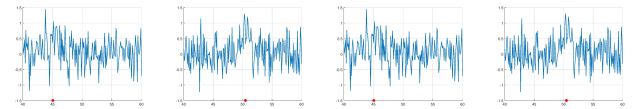


Fig. 6. The 1-D image measurements for  $t = 25, 50, 75, 100$  time steps from left to right panels. Along the x-axis is the surveillance region and along the y-axis is the intensity. The red marker is the actual target position.

state transition noise variance is  $\tau^2 = 0.2$ . The WSLAPF threshold is  $T_h = 0.001$ . The reported results are averaged over 500 Monte Carlo runs. Fig. 7 shows the RMSE versus the error precision. It can be observed that the proposed method tracks fairly accurately in low noise conditions and becomes inaccurate in high noise conditions. This could be due to more small weight samples being included into the particle lot. However it can be seen that the error is not substantially worse compared to the conventional PFs.

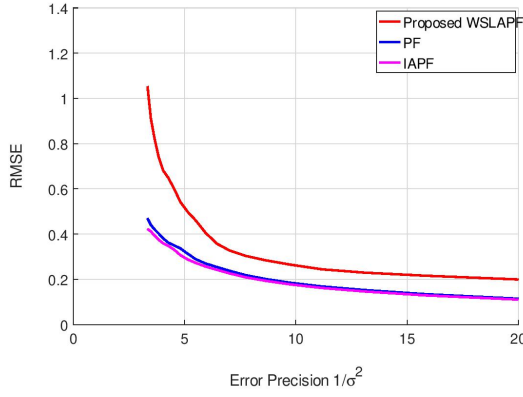


Fig. 7. The RMSE versus the error precision  $1/\sigma^2$ .

Fig. 8 shows the TxRMSE versus the error precision. It can be observed that poor track performance of the proposed WSLAPF in high noise conditions is offset by its fast computation. Now its performance is comparable to the standard PF and is nearly 3.5 times superior to the IAPF at  $\sigma^2 = 1/5$ .

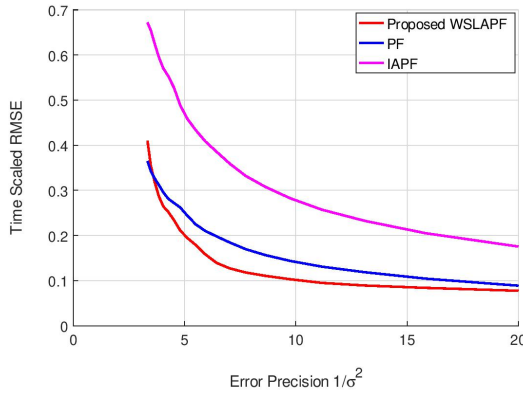


Fig. 8. The time-scaled-RMSE versus the error precision  $1/\sigma^2$ .

The key challenges in the proposed WSLPAF observed in this evaluation study is that the choice of the threshold  $T_h$  becomes critical for accurate tracking. A detailed study on this choice will be done in the future. The key merits of our proposal are accurate tracking by virtue of sampling from regions of high importance (weights) and high speed computation by virtue of avoiding the resampling step. These merits are numerically shown in the TxRMSE plots for the two nonlinear models.

## VI. CONCLUSION

In this paper, we presented a novel lookahead strategy for the particle filter. Here the previous set of particles are propagated forward in time by leveraging the incoming observation in the propagation step via a sorting and deterministic selection scheme. The proposal ensures particles to lie in regions of high importance and hence aids in accurate tracking. The proposal

also avoids the need to resample thus being computationally efficient. The tracking accuracy and time efficiency of our proposal are evaluated using numerical simulations.

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